Interaction Notes

Note 504

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Concerning the Identification of Buried Dielectric Targets

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Abstract

For identifying dielectric targets buried in a lossy dielectric such as soil, one can use the natural frequencies as a signature. However, these are dependent not only on the constitutive parameters of the target, but also on those of the surrounding medium. For frequencies above the relaxation frequency of the medium (the high-frequency window) and for the relative dielectric constant ϵ_r of the target (relative to the external medium) sufficiently small one can evaluate the natural frequencies as perturbations based on the asymptotics for $\epsilon_r \to 0$. This gives a set of external resonances dominated by the external medium parameters, and a set of internal resonances dominated by the internal medium parameters. The latter are cavity resonances with damping from the external medium proportional to $\epsilon_r^{1/2}$.

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I. Introduction

Identification of buried targets using electromagnetic fields relies fundamentally on the differences of the constitutive parameters of the buried target from those of the surrounding medium. Previous papers have considered the identification of such targets based on the singularity expansion method (SEM) involving complex natural frequencies s_α and various properties of the pole residues. Perfectly conducting targets have their SEM parameters scale in a simple way based on their free-space values [5]. Highly conducting targets have a set of negative real s_α based on the low-frequency diffusion of magnetic fields in the target [6]; the magnetic polarizability dyadic has real eigenvectors which are aligned according to any symmetry planes and axes the target may have [7].

The present paper considers another class of buried targets: dielectric targets as in fig. 1.1. In this case the permeability of the target and the surrounding medium are assumed to the same, i.e., μ_0 , the permeability of free space. The conductivity of the target is assumed zero, while that of the surrounding medium, σ_1 , is assumed non-zero, such as might characterize typical soils (say 10^{-3} to 10^{-2} S/m), sea water (4 S/m), etc. Here is a difference one might exploit by application of a DC electric potential between earthed conductors and observing the variation of the electric field tangential to the ground surface. However, other items such as rocks can give similar perturbation of the static electric field.

Assuming that the permitivity ϵ_2 of the target is succinctly different from ϵ_1 of the surrounding medium, then one may also consider the complex resonances s_α which occur for wavelengths of the order of the target size (or less) in the target as well as in the surrounding medium. This can be a complicated problem but one worth understanding due to its potential significance for target identification. Define

$$\epsilon_r \equiv \frac{\epsilon_2}{\epsilon_1} \tag{1.1}$$

as the relative dielectric constant of the target referenced to the external medium. With ϵ_2 as about $2\epsilon_0$ or $3\epsilon_0$ and typical soil permittivities (in the 100 MHz regime) as about $10\epsilon_0$, then one might think of ϵ_r as small compared to 1.0 and approximate accordingly. If the external medium is water then ϵ_1 is about $81\epsilon_0$ and the approximation is even better. In soil the water content also has an important influence on ϵ_1. Of course, one can consider the opposite approximation of $\epsilon_r >> 1$, but this may not be of as great practical significance. For the present discussion the dielectric target is also considered uniform and isotropic; this is a case of interest and is a logical step before considering inhomogeneous and/or an isotropic dielectrics.

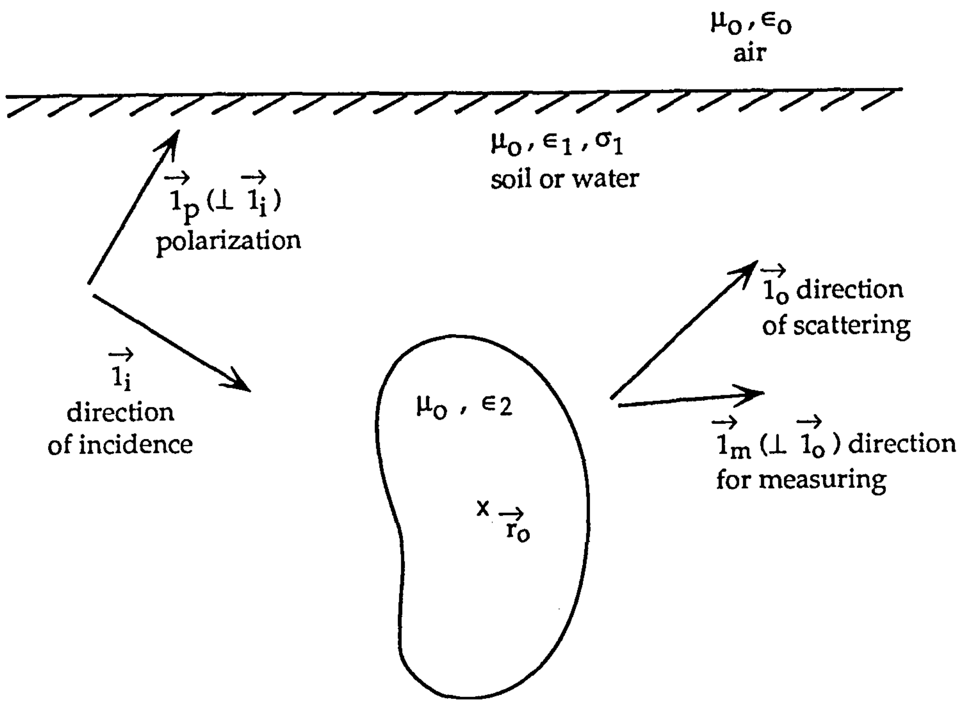


Fig. 1.1 Buried Dielectric Target

The important parameters for wave propagation in the two media are the propagation constants

$$\begin{aligned}
\tilde{\gamma}_1 &= \left[s\,\mu_o[\sigma_1 + s\,\epsilon_1]\right]^{\frac{1}{2}} = s\sqrt{\mu_o\,\epsilon_1}\left[1+\frac{\sigma_1}{s\,\epsilon_1}\right]^{\frac{1}{2}} \\
&\equiv \text{propagation constant in external medium} \\
\tilde{\gamma}_2 &= s\sqrt{\mu_o\,\epsilon_2} \\
&\equiv \text{propagation constant in target} \\
\sim &\equiv \text{Laplace transform (two-sided) with respect to time} \\
s &= \Omega + j\omega \equiv \text{complex frequency} \\
&\equiv \text{Laplace-transform variable}
\end{aligned} \tag{1.2}$$

and wave impedances

$$\begin{aligned}
\tilde{Z}_1 &= \left[\frac{s\,\mu_o}{\sigma_1 + s\,\epsilon_1}\right]^{\frac{1}{2}} = \sqrt{\frac{\mu_o}{\epsilon_1}}\left[1 + \frac{\sigma_1}{s\,\epsilon_1}\right]^{-\frac{1}{2}} \\
&\equiv \text{wave impedance in external medium} \\
Z_2 &= \sqrt{\frac{\mu_o}{\epsilon_2}} \\
&\equiv \text{wave impedance in target}
\end{aligned} \tag{1.3}$$

These have convenient combinations as

$$\begin{aligned}
\tilde{\gamma}_1\,\tilde{Z}_1 &= s\mu_o = \tilde{\gamma}_2\,Z_2 \\
\frac{\tilde{\gamma}_1}{\tilde{Z}_1} &= \sigma_1 + s\,\epsilon_1 \\
\frac{\tilde{\gamma}_2}{Z_2} &= s\,\epsilon_2
\end{aligned} \tag{1.4}$$

Note that differences between the two media do not appear in the products but rather in the ratios above. For later use we also have

$$\tilde{\xi} \equiv \frac{Z_2}{\tilde{Z}_1} = \frac{\tilde{\gamma}_1}{\tilde{\gamma}_2} = \epsilon_r^{-\frac{1}{2}}\left[1 + \frac{\sigma_1}{s\,\epsilon_1}\right]^{\frac{1}{2}} \tag{1.5}$$

and note that small ϵ_r implies large $\left|\tilde{\xi}(j\omega)\right|$.

So now consider some appropriate approximations that we can use later in canonical problems. Small ϵ_r, corresponding to large $\tilde{\xi}$ or small $\tilde{Z}_1$, can be thought of as a perturbation from the condition of an electric boundary (perfectly conducting sheet) around the target (for consideration of the internal resonances). (Conversely small $\tilde{\xi}$ corresponds to the case of a magnetic boundary around the target.) So we can start from the simpler case of a lossless cavity and see how the external medium changes (especially dampens) these resonances to positions away from the $j\omega$ axis.

There is also the consideration of the relative propagation speeds in the two media as

$$
\begin{aligned}
\tilde{v}_1 &\equiv \frac{s}{\tilde{\gamma}_1} = \left[\mu_o \epsilon_1\right]^{-\frac{1}{2}} \left[1 + \frac{\sigma_1}{s\epsilon_1}\right]^{-\frac{1}{2}} \\
v_2 &\equiv \frac{s}{\tilde{\gamma}_2} = \left[\mu_o \epsilon_2\right]^{-\frac{1}{2}} \\
\frac{v_2}{\tilde{v}_1} &= \frac{\tilde{\gamma}_1}{\tilde{\gamma}_2} = \tilde{\xi} = \epsilon_r^{-\frac{1}{2}} \left[1 + \frac{\sigma_1}{s\epsilon_1}\right]^{\frac{1}{2}}
\end{aligned}
\tag{1.6}
$$

For the case of relatively small $\tilde{v}_1$ (small ϵ_r) we have the well-known Brewster phenomenon for transmission of waves from the target to the external medium (total transmission if $\sigma_1 = 0$) provided the wave in the target is E (or TM) polarized with respect to the boundary ($\vec{H}$ parallel to the boundary). For such waves propagating with angles of incidence (with respect to the boundary) near the Brewster angle there is negligible reflection of the wave back into the target to maintain the resonance and extremely high damping occurs. One then might expect H (or TE) conditions for waves at the boundary to be associated with resonant modes that are in general less damped by the external medium (again, for small ϵ_r).

With these general physical considerations in mind let us now consider some specific geometries to see what they can tell us.

II. Infinite Dielectric Slab at Normal Incidence

Consider the canonical geometry of an infinite dielectric slab of thickness ℓ, embedded in a lossy dielectric medium as in fig. 2.1. Strictly speaking such an infinite slab does not have natural frequencies since the pole locations are functions of the direction of incidence. Of course we are concerned with finite-size targets, so let us consider the case of normal incidence as an approximate way to model some of the poles of a slab with finite dimensions (albeit large compared to ℓ) transverse to the direction of incidence.

This case of normal incidence is like a transmission-line calculation with the electric field taking the role of voltage and the magnetic field that of current. The polarization $\vec{1}_p$ of the electric field is parallel to the two interfaces with

$$\begin{aligned} &\vec{1}_p \cdot \vec{1}_z = 0 \\ &\vec{1}_z \equiv \text{direction of incidence} \end{aligned} \tag{2.1}$$

The magnetic field is polarized in the direction $\vec{1}_z \times \vec{1}_p$ for right (+z) propagating waves. Note that this TEM incident wave has both electric and magnetic fields parallel to the surfaces of the scatterer. As such it is a special case which is both *TE* and *TM*.

At $z = \ell$ we have

$$\begin{aligned} \tilde{T}_\ell &= \frac{2\tilde{Z}_1}{\tilde{Z}_1 + Z_2} = \frac{\tilde{E}_t(s)}{\tilde{E}_1(s)} e^{[\tilde{\gamma}_2 - \tilde{\gamma}_1]\ell} = 1 + \tilde{R}_\ell \\ \tilde{R}_\ell &= \frac{\tilde{Z}_1 - Z_2}{\tilde{Z}_1 + Z_2} = \frac{1 - \tilde{\xi}}{1 + \tilde{\xi}} = \frac{\tilde{E}_2(s)}{\tilde{E}_1(s)} e^{2\tilde{\gamma}_2 \ell} \end{aligned} \tag{2.2}$$

With this, go now to the first interface at $z = 0$ and obtain

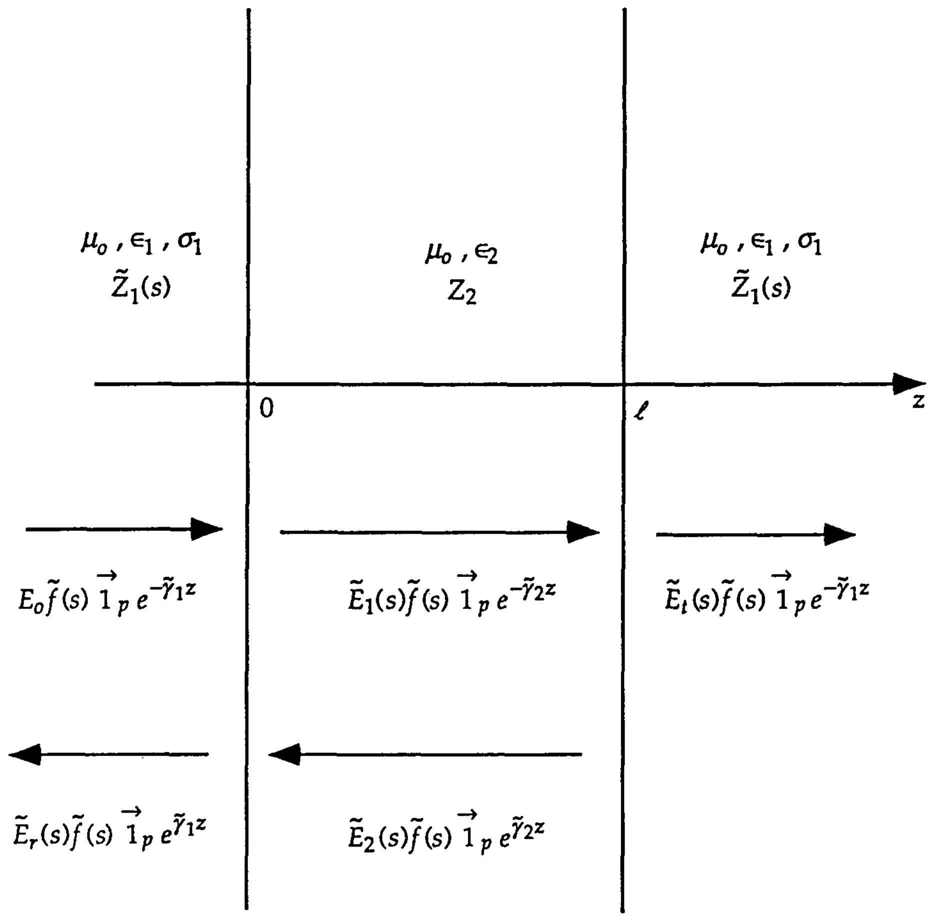


Fig. 2.1 Infinite Dielectric Slab in External Lossy Dielectric with Normal Wave Incidence

$$\tilde{R}_0 \equiv \frac{\tilde{E}_r}{E_o} = \frac{\tilde{Z}_{in} - \tilde{Z}_1}{\tilde{Z}_{in} + \tilde{Z}_1}$$

$$\tilde{Z}_{in} \equiv \frac{\tilde{E}_1 + \tilde{E}_2}{\frac{\tilde{E}_1}{Z_2} - \frac{\tilde{E}_2}{Z_2}} = Z_2 \frac{1+\tilde{R}_\ell\; e^{-2\tilde{\gamma}_2\ell}}{1-\tilde{R}_\ell\; e^{-2\tilde{\gamma}_2\ell}} \tag{2.3}$$

$$= Z_2 \frac{\cosh(\tilde{\gamma}_2\ell) + \tilde{\xi}\; \sinh(\tilde{\gamma}_2\ell)}{\sinh(\tilde{\gamma}_2\ell) + \tilde{\xi}\; \cosh(\tilde{\gamma}_2\ell)}$$

from which we find the reflection or backscattering coefficient as

$$\tilde{R}_0 = \frac{\frac{\tilde{Z}_{in}}{Z_2}\tilde{\xi} - 1}{\frac{\tilde{Z}_{in}}{Z_2}\tilde{\xi} + 1}$$

$$= \frac{\left[\tilde{\xi}^2 - 1\right]\sinh(\tilde{\gamma}_2\ell)}{\tilde{2}\;\tilde{\xi}\;\cosh(\tilde{\gamma}_2\ell) + \left[\tilde{\xi}^2 + 1\right]\sinh(\tilde{\gamma}_2\ell)} \tag{2.4}$$

$$\tilde{R}_0 \to \begin{cases} -1 & \text{as } \tilde{\xi} \to 0 \\ 0 & \text{as } \tilde{\xi} \to 1 \\ 1 & \text{as } \tilde{\xi} \to \infty \end{cases}$$

The incident wave has an arbitrary waveform $\tilde{f}(s)$ (or $f(t)$) which appears in all subsequent waves, but is not important for present considerations.

The poles (indicated by argument $s = s_\alpha$) are at

$$0 = \frac{\tilde{\xi}^2(s_\alpha)+1}{2\,\tilde{\xi}(s_\alpha)} + \coth(\tilde{\gamma}_2(s_\alpha)\ell) \tag{2.5}$$

For large $\tilde{\xi}$ or small ϵ_r there is a limiting form given by

$$\begin{aligned} T_\ell &\equiv \sqrt{\mu_o\,\epsilon_2}\;\ell \\ 0 &= \sinh\left(\tilde{\gamma}_2\left(s_\alpha^{(o)}\right)\ell\right) = \sinh\left(s_\alpha^{(o)}\,T_\ell\right) \\ 0 &= \sin\left(\omega_\alpha^{(o)}T_\ell\right) \\ s_\alpha^{(o)} &= j\omega_\alpha^{(o)} \\ \omega_\alpha^{(o)}\,T_\ell &= n\pi \;\text{ for }\; n = 1, 2, \ldots \end{aligned} \tag{2.6}$$

Note that, as a limiting case,

$$\tilde{R}_0\left(s_\alpha^{(o)}\right) = 0 \tag{2.7}$$

Continuing the expansion let

$$s_\alpha \equiv s_\alpha^{(o)} + \Delta s_\alpha \tag{2.8}$$

and note that

$$\begin{aligned} \cosh\left(s_\alpha^{(o)} T_\ell\right) &= \cos\left(\omega_\alpha^{(o)} T_\ell\right) = \cos(n\pi) \\ &= (-1)^n \\ \sinh\left(s_\alpha T_\ell\right) &= (-1)^n \sinh\left(\Delta s_\alpha T_\ell\right) \\ \cosh\left(s_\alpha T_\ell\right) &= (-1)^n \cosh\left(\Delta s_\alpha T_\ell\right) \end{aligned} \tag{2.9}$$

Then rewrite (2.5) as

$$\begin{aligned} 0 &= \frac{\tilde{\xi}^2(s_\alpha)+1}{\tilde{\xi}(s_\alpha)} + \coth\left(\Delta s_\alpha T_\ell\right) \\ \Delta s_\alpha T_\ell &= -\operatorname{arctanh}\left[\frac{\tilde{\xi}(s_\alpha)}{\tilde{\xi}^2(s_\alpha)+1}\right] \end{aligned} \tag{2.10}$$

Expand for small arguments (large $\tilde{\xi}$) as

$$\begin{aligned} \Delta s_\alpha T_\ell &= -\frac{\tilde{\xi}(s_\alpha)}{\tilde{\xi}^2(s_\alpha)+1} + O\left(\tilde{\xi}^{-3}(s_\alpha)\right) \\ &= -\tilde{\xi}^{-1}(s_\alpha) + O\left(\tilde{\xi}^{-3}(s_\alpha)\right) \text{ as } \tilde{\xi}(s_\alpha) \to \infty \end{aligned} \tag{2.11}$$

Now write

$$\tilde{\xi}(s_\alpha) = \tilde{\xi}\left(s_\alpha^{(o)}\right)\left[1 + O\left(\Delta s_\alpha\right)\right] \text{ as } \Delta s_a \to 0 \tag{2.12}$$

giving

$$\begin{aligned}\Delta s_\alpha T_\ell &= -\tilde{\xi}^{-1}\left(s_\alpha^{(o)}\right) + O\left(\left(\Delta s_\alpha T_\ell\right)^2\right) + O\left(\tilde{\xi}^{-3}\left(s_\alpha^{(o)}\right)\right) \\ &\qquad \text{as } \tilde{\xi}\left(s_\alpha^{(o)}\right) \to \infty \text{ and } \Delta s_\alpha T_\ell \to 0 \\ &= -\tilde{\xi}^{-1}\left(s_\alpha^{(o)}\right) + O\left(\tilde{\xi}^{-2}\left(s_\alpha^{(o)}\right)\right) \text{ as } \tilde{\xi}(s_\alpha) \to \infty\end{aligned} \tag{2.13}$$

Consider now the effect of the external medium on these pole locations. Recall from (1.5)

$$\tilde{\xi}^{-1}\left(s_\alpha^{(o)}\right) = \epsilon_r^{\frac{1}{2}}\left[1 + \frac{\sigma_1}{s_\alpha^{(o)}\epsilon_1}\right]^{-\frac{1}{2}} \tag{2.14}$$

Provided that frequencies of interest are in the high-frequency window [5,10], this implies

$$\begin{aligned}&\left|\frac{s_\alpha^{(o)}\epsilon_1}{\sigma_1}\right| >> 1 \\ &\tilde{\xi}^{-1}\left(s_\alpha^{(o)}\right) = \epsilon_r^{\frac{1}{2}}\left[1 - \frac{1}{2}\frac{\sigma_1}{s_\alpha^{(o)}\epsilon_1} + O\left(\left(\frac{\sigma_1}{s_\alpha^{(o)}\epsilon_1}\right)^2\right)\right] \\ &\qquad \text{as } \frac{\sigma_1}{s_\alpha^{(o)}\epsilon_1} \to 0\end{aligned} \tag{2.15}$$

The limiting case of a pure dielectric external medium (σ_1 and ϵ_1 real) gives

$$\Delta s_\alpha T_\ell = -\epsilon_r^{\frac{1}{2}} + O(\epsilon_r) \text{ as } \epsilon_r \to 0 \tag{2.16}$$

showing a significant shift to the left (damping) in the s plane. Comparing to the unperturbed resonance in (2.6) gives

$$\frac{\Delta s_\alpha}{\omega_\alpha^{(o)}} = -\frac{\epsilon_r^{\frac{1}{2}}}{n\pi} + O(\epsilon_r) \text{ as } \epsilon_r \to 0 \tag{2.17}$$

For ϵ_r of order, say 0.25, the lowest resonance is shifted to the left by a relative amount $1/(2\pi)$. Including the conductivity of the external medium we have

$$\Delta s_\alpha T_\ell = -\epsilon_r^{\frac{1}{2}}\left[1+\frac{j}{2}\frac{\sigma_1}{\omega_\alpha^{(o)}\epsilon_1} + O\left(\left(\frac{\sigma_1}{\omega_\alpha^{(o)}\epsilon_1}\right)^2\right)\right] + O\left(\tilde{\xi}^{-3}(s_\alpha)\right)$$
$$\text{as } \frac{\sigma_1}{s_\alpha^{(o)}\epsilon_1} \to 0 \text{ and } \tilde{\xi}(s_\alpha) \to \infty \tag{2.18}$$

In this more general form we see an additional correction in that $\Delta s_\alpha T_\ell$ has a small negative imaginary part (with positive $\omega_\alpha^{(o)}$) moving the ω part (imaginary part) of the poles closer to the origin.

Looking at these results we find that the imaginary part of these poles is not changed much by the external medium (for small ϵ_r), provided that the poles occur at frequencies large compared to σ_1/ϵ_1. The real part (damping) of the pole is, however, significantly affected by ϵ_r, the amount of damping being a measure of ϵ_r in a simple (but approximate) scaling relationship.

For completeness the reflection coefficient can be represented near one of these poles by

$$\tilde{R}_0 = \frac{\eta_\alpha}{s - s_\alpha} \text{ as } s \to s_\alpha$$
$$\eta_\alpha \equiv \text{coupling coefficient} \tag{2.19}$$

For small Δs_α in (2.8) we have from (2.4)

$$\eta_\alpha = \frac{\left[\tilde{\xi}^2(s_\alpha)-1\right]\sinh(s_\alpha T_\ell)}{\frac{d}{ds}\left[2\tilde{\xi}(s)\cosh(sT_\ell)+\left[\tilde{\xi}^2(s)+1\right]\sinh(sT_\ell)\right]\Big|_{s=s_\alpha}}$$
$$= \frac{\left[\tilde{\xi}\left(s_\alpha^{(o)}\right)-1\right]\Delta s_\alpha T_\ell}{2\frac{d\tilde{\xi}\left(s_\alpha^{(o)}\right)}{d\left(s_\alpha^{(o)}\right)} + \left[\tilde{\xi}^2\left(s_\alpha^{(o)}\right)+1\right]T_\ell} + O\left((\Delta s_\alpha)^2\right)$$
$$\text{as } \Delta s_\alpha \to 0 \tag{2.20}$$

Neglecting the conductivity σ_1 of the external medium so that $\tilde{\xi}$ is not a function of s gives

$$\eta_\alpha = \frac{\tilde{\xi}^2-1}{\tilde{\xi}^2+1}\Delta s_\alpha + O\left((\Delta s_\alpha)^2\right)$$
$$= \frac{1-\epsilon_r}{1+\epsilon_r}\Delta s_\alpha + O\left((\Delta s_\alpha)^2\right) \text{ as } \Delta s_\alpha \to 0 \tag{2.21}$$

with Δs_α as in (2.16).

III. Dielectric Sphere

As indicated in fig. 3.1, consider a spherical scatterer (radius a) with properties of the two media as discussed in Section I. Taking the results from [4] we have an incident plane wave in the usual spherical (r , θ , ϕ) coordinates as

$$\begin{aligned}
\vec{\tilde{E}}^{(inc)}(\vec{r},s) &= E_o\,\tilde{f}(s)\,\vec{1}_x\,e^{-\tilde{\gamma}_1 z} \\
&= E_o\,\tilde{f}(s)\sum_{n=1}^{\infty}(-1)^n\frac{2n+1}{n(n+1)}\left[-\vec{M}^{(1)}_{n,1,o}(\tilde{\gamma}_1\,\vec{r}) + \vec{N}^{(1)}_{n,1,e}(\tilde{\gamma}_1\,\vec{r})\right] \\
\vec{\tilde{H}}^{(inc)}(\vec{r},s) &= \frac{E_o}{\tilde{Z}_1(s)}\tilde{f}(s)\,\vec{1}_y\,e^{-\tilde{\gamma}_1 z} \\
&= \frac{E_o}{\tilde{Z}_1(s)}\tilde{f}(s)\sum_{n=1}^{\infty}(-1)^n\frac{2n+1}{n(n+1)}\left[\vec{M}^{(1)}_{n,1,e}(\tilde{\gamma}_1\,\vec{r}) + \vec{N}^{(1)}_{n,1,o}(\tilde{\gamma}_1\,\vec{r})\right]
\end{aligned} \tag{3.1}$$

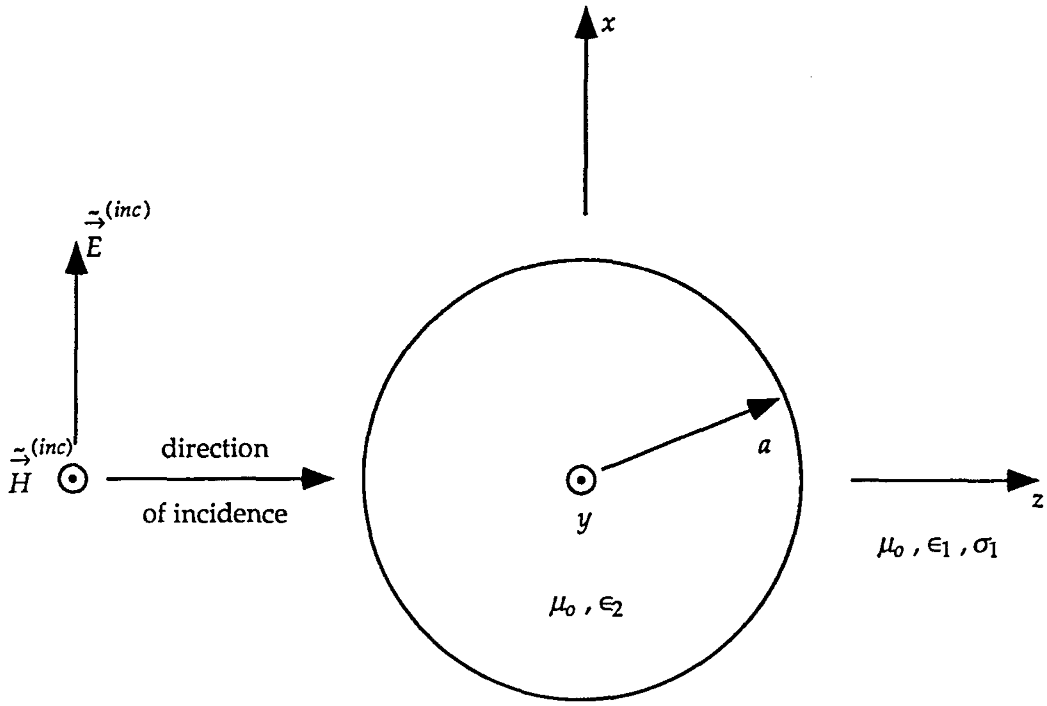
The vector wave functions are

$$\begin{aligned}
\vec{M}^{(\ell)}_{n,m,\sigma}(\tilde{\gamma}\,\vec{r}) &= f_n^{(\ell)}(\gamma r)\;\vec{R}_{n,m,\sigma}(\theta,\phi) \\
&= -\frac{1}{\gamma}\nabla\times\vec{N}^{(\ell)}_{n,m,\sigma}(\gamma\,\vec{r}) \\
\vec{N}^{(\ell)}_{n,m,\sigma}(\gamma\,\vec{r}) &= n(n+1)\frac{f_n^{(\ell)}(\gamma r)}{\gamma r}\;\vec{P}_{n,m,\sigma}(\theta,\phi) + \frac{\left[\gamma r\, f_n^{(\ell)}(\gamma r)\right]'}{\gamma r}\;\vec{Q}_{n,m,\sigma}(\theta,\phi) \\
&= \frac{1}{\gamma}\nabla\times\vec{M}^{(\ell)}_{n,m,\sigma}(\gamma\,\vec{r})
\end{aligned} \tag{3.2}$$

the spherical harmonics are

$$\begin{aligned}
Y_{n,m,\sigma}(\theta,\phi) &= Y_{n,m,{e \atop o}}(\theta,\phi) = P_n^{(m)}(\cos(\theta))\begin{Bmatrix}\cos(m\phi)\\ \sin(m\phi)\end{Bmatrix} \\
\vec{P}_{n,m,\sigma}(\theta,\phi) &= Y_{n,m,\sigma}(\theta,\phi)\,\vec{1}_r \\
\vec{Q}_{n,m,\sigma}(\theta,\phi) &= \nabla_{\theta,\phi}Y_{n,m,\sigma}(\theta,\phi) = \vec{1}_r\times\vec{R}_{n,m,\sigma}(\theta,\phi) \\
\vec{R}_{n,m,\sigma}(\theta,\phi) &= \nabla_{\theta,\phi}\times\vec{P}_{n,m,\sigma}(\theta,\phi) = -\vec{1}_r\times\vec{Q}_{n,m,\sigma}(\theta,\phi)
\end{aligned} \tag{3.3}$$

and the modified spherical Bessel functions are



cylindrical coordinates : $x = \Psi\cos(\phi)$, $y = \Psi\sin(\phi)$

spherical coordinates : $z = r\cos(\theta)$, $\Psi = r\sin(\theta)$

Fig. 3.1 Dielectric Sphere in External Lossy Dielectric

$$
\begin{aligned}
f_n^{(1)}(\zeta) &= i_n(\zeta) \text{ analytic at } \zeta = 0 \text{ for incident wave} \\
f_n^{(2)}(\zeta) &= k_n(\zeta) \text{ for outgoing wave} \\
f_n^{(3)}(\zeta) &= k_n(-\zeta) \text{ for incoming wave}
\end{aligned}
\tag{3.4}
$$

where a prime is used to indicate a derivative with respect to the argument of these functions. Additional details concerning these functions are found in [4].

Similarly expand the scattered fields (medium 1) as

$$
\begin{aligned}
\tilde{\vec{E}}^{(sc)}(\vec{r},s) &= E_o \tilde{f}(s) \sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n(n+1)} \left[-\tilde{a}_n \vec{M}^{(2)}_{n,1,o}(\tilde{\gamma}_1 \vec{r}) + \tilde{b}_n \vec{N}^{(2)}_{n,1,e}(\tilde{\gamma}_1 \vec{r}) \right] \\
\tilde{\vec{H}}^{(sc)}(\vec{r},s) &= \frac{E_o}{\tilde{Z}_1(s)} \tilde{f}(s) \sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n(n+1)} \left[\tilde{b}_n \vec{M}^{(2)}_{n,1,e}(\tilde{\gamma}_1 \vec{r}) + \tilde{a}_n \vec{N}^{(2)}_{n,1,o}(\tilde{\gamma}_1 \vec{r}) \right]
\end{aligned}
\tag{3.5}
$$

The fields internal to the target (medium 2) take the form

$$
\begin{aligned}
\tilde{\vec{E}}^{(sc)}(\vec{r},s) &= E_o \tilde{f}(s) \sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n(n+1)} \left[-\tilde{c}_n \vec{M}^{(1)}_{n,1,o}(\tilde{\gamma}_2 \vec{r}) + \tilde{d}_n \vec{N}^{(1)}_{n,1,e}(\tilde{\gamma}_2 \vec{r}) \right] \\
\tilde{\vec{H}}^{(sc)}(\vec{r},s) &= \frac{E_o}{\tilde{Z}_2(s)} \tilde{f}(s) \sum_{n=1}^{\infty} (-1)^n \frac{2n+1}{n(n+1)} \left[\tilde{d}_n \vec{M}^{(1)}_{n,1,e}(\tilde{\gamma}_2 \vec{r}) + \tilde{c}_n \vec{N}^{(1)}_{n,1,o}(\tilde{\gamma}_2 \vec{r}) \right]
\end{aligned}
\tag{3.6}
$$

Matching tangential electric field on r = a gives

$$
\begin{aligned}
& i_n(\tilde{\gamma}_1 a) + \tilde{a}_n k_n(\tilde{\gamma}_1 a) = c_n \; i_n(\tilde{\gamma}_2 a) \\
& \frac{\left[\tilde{\gamma}_1 a \; i_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a} + \tilde{b}_n \frac{\left[\tilde{\gamma}_1 a \; k_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a} = \tilde{d}_n \frac{\left[\tilde{\gamma}_2 a \; i_n(\tilde{\gamma}_2 a)\right]'}{\tilde{\gamma}_2 a}
\end{aligned}
\tag{3.7}
$$

Matching tangential magnetic field on r = a gives

$$
\begin{aligned}
& \frac{1}{\tilde{Z}_1(s)} \left[i_n(\tilde{\gamma}_1 a) + \tilde{b}_n k_n(\tilde{\gamma}_1 a) \right] = \frac{1}{Z_2} \tilde{d}_n \, i_n(\tilde{\gamma}_2 a) \\
& \frac{1}{\tilde{Z}_1(s)} \left[\frac{\left[\tilde{\gamma}_1 a \, i_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a} + \tilde{a}_n \frac{\left[\tilde{\gamma}_1 a \, k_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a} \right] = \frac{1}{Z_2} \tilde{c}_n \frac{\left[\tilde{\gamma}_2 a \, i_n(\tilde{\gamma}_2 a)\right]'}{\tilde{\gamma}_2 a}
\end{aligned}
\tag{3.8}
$$

Solving forst for $\tilde{c}_n$ and $\tilde{d}_n$ as

$$\begin{aligned}
\tilde{c}_n &= \frac{1}{i_n(\tilde{\gamma}_2 a)}\left[i_n(\tilde{\gamma}_1 a) + \tilde{a}_n\, k_n(\tilde{\gamma}_1 a)\right] \\
&= \frac{\tilde{\xi}\ \tilde{\gamma}_2 a}{\left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]'}\left[\frac{\left[\tilde{\gamma}_1 a\, i_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a} + \tilde{a}_n \frac{\left[\tilde{\gamma}_1 a\, k_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a}\right] \\
\tilde{d}_n &= \frac{\tilde{\xi}}{i_n(\tilde{\gamma}_2 a)}\left[i_n(\tilde{\gamma}_1 a) + \tilde{b}_n\, k_n(\tilde{\gamma}_1 a)\right] \\
&= \frac{\tilde{\gamma}_2 a}{\left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]'}\left[\frac{\left[\tilde{\gamma}_1 a\, i_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a} + \tilde{b}_n \frac{\left[\tilde{\gamma}_1 a\, k_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a}\right]
\end{aligned} \tag{3.9}$$

allows us to find the required coefficients $\tilde{a}_n$ and $\tilde{b}_n$ for the scattered field as

$$\begin{aligned}
\tilde{a}_n &= -\left[i_n(\tilde{\gamma}_1 a) - \tilde{\xi}\,\frac{\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)}{\left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]'}\,\frac{\left[\tilde{\gamma}_1 a\, i_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a}\right] \\
&\quad \left[k_n(\tilde{\gamma}_1 a) - \tilde{\xi}\,\frac{\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)}{\left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]'}\,\frac{\left[\tilde{\gamma}_1 a\, k_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a}\right]^{-1} \\
\tilde{b}_n &= -\left[i_n(\tilde{\gamma}_1 a) - \frac{\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)}{\tilde{\xi}\left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]'}\,\frac{\left[\tilde{\gamma}_1 a\, i_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a}\right] \\
&\quad \left[k_n(\tilde{\gamma}_1 a) - \frac{\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)}{\tilde{\xi}\left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]'}\,\frac{\left[\tilde{\gamma}_1 a\, k_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a}\right]^{-1}
\end{aligned} \tag{3.10}$$

It is the poles of these coefficients which are the poles of the scattered fields in (3.5). Note that the $\tilde{a}_n$ correspond to H (or TE) modes, and the $\tilde{b}_n$ correspond to E (or TM) modes.

Consider the special case of $\epsilon_r = 0$ or $\tilde{\xi} = \infty$ The interior modes are those of a dielectric sphere surrounded by an electric (or perfectly conducting) boundary on r = a. Viewed from the exterior medium the dielectric sphere appears to present an infinite sheet impedance (magnetic boundary) to the external waves. So we can think of the scattering poles as being of two kinds, external and internal. For the external poles then away from the roots of $(\tilde{\gamma}_2 a)$ and $\left[\tilde{\gamma}_2 a\ (\tilde{\gamma}_2 a)\right]'$ we have

$$
\begin{aligned}
\tilde{a}_n &= -\frac{\left[\tilde{\gamma}_1 a \; i_n(\tilde{\gamma}_1 a)\right]'}{\left[\tilde{\gamma}_1 a \; k_n(\tilde{\gamma}_1 a)\right]'} \qquad \text{as } \tilde{\xi} \to \infty \\
\tilde{b}_n &= -\frac{i_n(\tilde{\gamma}_1 a)}{k_n(\tilde{\gamma}_1 a)} \qquad \text{as } \tilde{\xi} \to \infty
\end{aligned}
\tag{3.11}
$$

Defining a parameter

$$
\Gamma^{(1)} \equiv \tilde{\gamma}_1 a = \left[s\mu_o(\sigma_1 + s\epsilon_1)\right]^{\frac{1}{2}} a \tag{3.12}
$$

then we have the natural frequencies associated with the roots

$$
\begin{aligned}
&\left[\Gamma_\alpha^{(1,H)} K_n\left(\Gamma_\alpha^{(1,H)}\right)\right]' = 0 \\
&K_n\left(\Gamma_\alpha^{(1,E)}\right) = 0 \\
&\Gamma_\alpha^{(1,H)} \equiv \text{roots for external } H \text{ (or } TE\text{) modes} \\
&\Gamma_\alpha^{(1,E)} \equiv \text{roots for external } E \text{ (or } TM\text{) modes}
\end{aligned}
\tag{3.13}
$$

Note that the roots for H modes correspond to those for E-modes for scattering from a perfectly conducting sphere, and similarly E modes correspond to H modes for scattering from a perfectly conducting sphere [2,3}. Having determined these roots, then (3.12) can be used to determine the external natural frequencies $s_\alpha^{(1)}$. This is the same scaling relationship as in [5] showing how these natural frequencies depend on the external-medium parameters. With the results from the perfectly conducting sphere well known [2,3], these external natural frequencies are known to be highly damped, i.e. significantly to the left in the s plane. For non-zero $\tilde{\xi}$, this is of course only an approximation.

The interior natural frequencies are found by reorganizing (3.10) as

$$
\begin{aligned}
\tilde{a}_n = -&\left[\frac{1}{\tilde{\xi}}\left[\tilde{\gamma}_2 a \, i_n(\tilde{\gamma}_2 a)\right]' i_n(\tilde{\gamma}_2 a) - \tilde{\gamma}_2 a \; i_n(\tilde{\gamma}_2 a)\frac{\left[\tilde{\gamma}_1 a \; i_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a}\right] \\
&\left[\frac{1}{\tilde{\xi}}\left[\tilde{\gamma}_2 a \, i_n(\tilde{\gamma}_2 a)\right]' k_n(\tilde{\gamma}_1 a) - \tilde{\gamma}_2 a \; i_n(\tilde{\gamma}_2 a)\frac{\left[\tilde{\gamma}_1 a \; k_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a}\right]^{-1}
\end{aligned}
$$

$$
\tilde{b}_n = -\left[\left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]' i_n(\tilde{\gamma}_1 a) - \frac{\tilde{\gamma}_2 a\ i_n(\tilde{\gamma}_2 a)}{\tilde{\xi}} \frac{\left[\tilde{\gamma}_1 a\ i_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a} \right]
\left[\left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]' K_n(\tilde{\gamma}_1 a) - \frac{\tilde{\gamma}_2 a\ i_n(\tilde{\gamma}_2 a)}{\tilde{\xi}} \frac{\left[\tilde{\gamma}_1 a\ k_n(\tilde{\gamma}_1 a)\right]'}{\tilde{\gamma}_1 a} \right]^{-1} \tag{3.14}
$$

Keeping away from $s = 0$, note that the interior resonances are on the $j\omega$ axis away from the roots of the functions of $\tilde{\gamma}_1 a$ which are in the left half plane (the exterior resonances). Letting $\tilde{\xi}$ be large (but not infinite) we see that the roots of the two denominators above are not cancelled by zeros of the numerators. Define a parameter

$$
\Gamma^{(2)} \equiv \tilde{\gamma}_2 a = s\sqrt{\mu_o \in_2} \tag{3.15}
$$

for the internal resonances. Now let $\tilde{\xi} \to \infty$ to give the roots

$$
\begin{aligned}
& i_n\left(\Gamma_\alpha^{(2,H)}\right) = 0 \\
& \left[\Gamma_\alpha^{(2,E)}\, i_n\left(\Gamma_\alpha^{(2,E)}\right)\right]' = 0 \\
& \Gamma_\alpha^{(2,E)} \equiv \text{roots for internal } H \text{ (or } TE\text{) modes} \\
& \Gamma_\alpha^{(2,E)} \equiv \text{roots for internal } E \text{ (or } TM\text{) modes}
\end{aligned} \tag{3.16}
$$

For these internal resonances the boundary condition on r = a is approximately that of a perfect electric conductor (in contradistinction to the case of the external resonances). As such these Γ_α s are imaginary numbers corresponding to s_α on the $j\omega$ axis.

Since $\tilde{\xi}$ is finite in practice the natural frequencies are perturbed from their positions in the s plane established by (3.13) and (3.16) and the constitutive parameters of the appropriate medium. The exterior resonances in (3.13) already exhibit damping, but the exterior resonances in (3.16) are undamped. So consider the damping of the interior resonances via a perturbation analysis of the poles in (3.14). Note the Wronskian relations [2,4,9]

$$
\begin{aligned}
& W\{i_n(\zeta), k_n(\zeta)\} \equiv i_n(\zeta)\, k_n'(\zeta) - i_n'(\zeta)\, k_n(\zeta) = -\zeta^{-2} \\
& W\{\zeta\, i_n(\zeta), \zeta\, k_n(\zeta)\} \equiv \zeta\, i_n(\zeta)\left[\zeta\, k_n(\zeta)\right]' - \left[\zeta\, i_n(\zeta)\right]' \zeta\, k_n(\zeta) = -1
\end{aligned} \tag{3.17}
$$

the Riccati-Bessel equation

$$\frac{\zeta^2}{\zeta^2+n(n+1)}\left[\zeta f_n^{(\ell)}(\zeta)\right]'' - \zeta f_n^{(\ell)}(\zeta) = 0 \tag{3.18}$$

and the large argument approximations

$$\begin{aligned}
\left[\zeta K_n(\zeta)\right] &= e^{-\zeta}\left[1+O\left(\zeta^{-1}\right)\right] \text{ as } \zeta \to \infty \\
\left[\zeta K_n(\zeta)\right]' &= -e^{-\zeta}\left[1+O\left(\zeta^{-1}\right)\right] \text{ as } \zeta \to \infty
\end{aligned} \tag{3.19}$$

Consider first the interior magnetic modes. Define

$$\begin{aligned}
T_a &\equiv \sqrt{\mu_o \in_2}\, a \\
\tilde{\gamma}_2 a &\equiv s\, T_a \\
s_\alpha^{(2,H,0)} T_a &\equiv \Gamma_\alpha^{(2,H)} \\
s_\alpha^{(2,H)} &\equiv s_\alpha^{(2,H,0)} + \Delta s_\alpha^{(2,H)}
\end{aligned} \tag{3.20}$$

so that we need the perturbation Δs_α of the interior natural frequencies due to finite $\tilde{\xi}$. Then from the denominator of $\tilde{a}_n$ in (3.14) write

$$\begin{aligned}
\gamma_2 a\, i_n(\gamma_2 a) &= \Delta s_\alpha^{(2,H)}\, T_a\left[\Gamma_\alpha^{(2,H)}\, i_n\left(\Gamma_\alpha^{(2,H)}\right)\right]' + O\left(\left(\Delta s_\alpha^{(2,H)}\right)^2\right) \text{ as } \Delta s_\alpha^{(2,H)} \to 0 \\
\Delta s_\alpha^{(2,H)}\, T_a &= \frac{1}{\tilde{\xi}}\frac{\gamma_1 a\, k_n(\gamma_1 a)}{\left[\gamma_1 a\, k_n(\gamma_2 a)\right]} + O\left(\left(\Delta s_\alpha^{(2,H)}\, T_a\right)^2\right) \text{ as } \Delta s_\alpha^{(2,H)} \to 0
\end{aligned} \tag{3.21}$$

For some chosen internal resonance $s_\alpha^{(2,H)}$, as $\tilde{\xi} \to \infty$ then $\tilde{\gamma}_1 a \to \infty$ from (1.5) and a large argument approximation of the Bessel functions is appropriate. From (3.19) we have

$$\begin{aligned}
\Delta s_\alpha^{(2,H)} T_a &= -\frac{1}{\tilde{\xi}} + O\left(\tilde{\xi}^{-2}\right) + O\left(\left(\Delta s_\alpha^{(2,H)} T_a\right)^2\right) \\
&\qquad \text{as } \tilde{\xi} \to \infty \text{ and } \Delta s_\alpha^{(2,H)} \to 0 \\
&= -\frac{1}{\tilde{\xi}} + O\left(\tilde{\xi}^{-2}\right) \text{as } \tilde{\xi} \to \infty
\end{aligned} \tag{3.22}$$

where $\tilde{\xi}$ is now evaluated at $s_\alpha^{(2,H,0)}$, the unperturbed resonance. Substituting for $\tilde{\xi}$ from (2.15) for $\sigma_1 = 0$ gives

$$\Delta s_{\alpha}^{(2,H)}\, T_a = -\epsilon_r^{\frac{1}{2}} + O(\epsilon_r) \tag{3.23}$$

just as for the slab in (2.16). Including σ_1 gives, as in (2.18), a small change in the $j\omega$ part.

For the interior electric modes define

$$\begin{aligned} s_{\alpha}^{(2,E,0)}\, T_a &\equiv \Gamma_{\alpha}^{(2,E)} \\ s_{\alpha}^{(2,E)} &\equiv s_{\alpha}^{(2,E,0)} + \Delta s_{\alpha}^{(2,E)} \end{aligned} \tag{3.24}$$

From the denominator of $\tilde{b}_n$ in (3.14) write

$$\begin{aligned} \left[\tilde{\gamma}_2 a\, i_n(\tilde{\gamma}_2 a)\right]' &= \Delta s_{\alpha}^{(2,E)}\, T_a \left[\Gamma_{\alpha}^{(2,E)}\, i_n\left(\Gamma_{\alpha}^{(2,E)}\right)\right]'' + O\left(\left(\Delta s_{\alpha}^{(2,E)}\right)^2\right) \text{ as } \Delta s_{\alpha}^{(2,E)} \to 0 \\ \Delta s_{\alpha}^{(2,E)}\, T_a &= \frac{1}{\tilde{\xi}} \frac{\Gamma_{\alpha}^{(2,E)}\, i_n\left(\Gamma_{\alpha}^{(2,E)}\right)}{\left[\Gamma_{\alpha}^{(2,E)}\, i_n\left(\Gamma_{\alpha}^{(2,E)}\right)\right]''} \frac{\left[\tilde{\gamma}_1 a\, k_n(\gamma_1 a)\right]'}{\gamma_1 a\, k_n(\gamma_1 a)} + O\left(\left(\Delta s_{\alpha}^{(2,E)}\, T_a\right)^2\right) \\ &= \frac{1}{\tilde{\xi}} \frac{\Gamma_{\alpha}^{(2,E)^2}}{\Gamma_{\alpha}^{(2,E)^2} + n(n+1)} \frac{\left[\gamma_1 a\, k_n(\gamma_1 a)\right]'}{\gamma_1 a\, k_n(\gamma_1 a)} + O\left(\left(\Delta s_{\alpha}^{(2,E)}\, T_a\right)^2\right) \text{ as } \Delta s_{\alpha}^{(2,E)} \to 0 \end{aligned} \tag{3.25}$$

where (3.18) has been used to simplify the result. For some chosen internal resonance $s_{\alpha}^{(2,E)}$ as $\tilde{\xi} \to \infty$ then $\tilde{\gamma}_1 a \to \infty$ and the Bessel functions are approximated to give

$$\begin{aligned} \Delta s_{\alpha}^{(2,E)}\, T_{\alpha} &= -\frac{1}{\tilde{\xi}} \frac{\Gamma_{\alpha}^{(2,E)^2}}{\Gamma_{\alpha}^{(2,E)^2} + n(n+1)} + O\left(\tilde{\xi}^{-2}\right) + O\left(\left(\Delta s_{\alpha}^{(2,E)}\, T_{\alpha}\right)^2\right) \\ &\qquad \text{as } \tilde{\xi} \to \infty \text{ and } \Delta s_{\alpha}^{(2,E)} \to 0 \\ &= -\frac{1}{\tilde{\xi}} \frac{\Gamma_{\alpha}^{(2,E)^2}}{\Gamma_{\alpha}^{(2,E)^2} + n(n+1)} + O\left(\tilde{\xi}^{-2}\right) \text{ as } \tilde{\xi} \to \infty \end{aligned} \tag{3.26}$$

where $\tilde{\xi}$ is now evaluated at $s_{\alpha}^{(2,H,0)}$, the unperturbed resonance. Substituting for $\tilde{\xi}$ from (2.15) for $\sigma_1 = 0$ gives

$$\Delta s_{\alpha}^{(2,E)}\, T_a = -\epsilon_r^{\frac{1}{2}} \frac{\Gamma_{\alpha}^{(2,E)^2}}{\Gamma_{\alpha}^{(2,E)^2} + n(n+1)} + O(\epsilon_r) \tag{3.27}$$

including σ_1 modifies this result as in (2. 18) with the inclusion of the new factor involving $\Gamma_\alpha^{(2,E)^2}$. Note that

$$\begin{gathered}\Gamma_\alpha^{(2,E)^2} < -n(n+1) \\ \frac{\Gamma_\alpha^{(2,E)^2}}{\Gamma_\alpha^{(2,E)^2}+1} > 1\end{gathered} \tag{3.28}$$

since all the roots are imaginary and larger than $n(n+1)$ in magnitude [1, 9]. Note that the lower order resonances (smaller $|\Gamma_\alpha^{(2,E)^2}|$) give more damping for the E modes, and the E-mode damping is greater than the H-mode damping. Furthermore, for a given n the lower order resonances have greater damping.

IV. General Properties of External Resonances

Considering the approximations concerning the dielectric sphere one can see some of the properties of a general dielectric target embedded in a medium of higher permittivity. For the external resonances the limit as $\tilde{\xi} \to \infty$ (or $\epsilon_r \to 0$) makes the surface S of the target (of volume V) behave as a perfect magnetic conductor (infinite surface impedance).

Letting S be a perfect magnetic conductor one next considers the dual problem in which the electric and magnetic fields are interchanged [8]. In the dual problem S is a perfect electric conductor. The propagation constant $\tilde{\gamma}_1(s)$ is unchanged in the transformation and it is certain characteristic values of $\tilde{\gamma}_1$ that correspond to the natural frequencies. As such the technique in [5] can now be applied. One can use the natural frequencies determined in free space (say $s_\alpha^{(f)}$) to datermine these propagation constants and then scale using the external medium parameters as

$$s_\alpha^{(f)} \sqrt{\mu_o \epsilon_o} \equiv \tilde{\gamma}_{f_\alpha} = \tilde{\gamma}_{1_\alpha} = \left[s_\alpha^{(1)} \mu_o \left[\sigma_1 + s_\alpha^{(1)} \right] \right]^{\frac{1}{2}} \tag{4.1}$$

From an experimental point of view one can then take a dielectric target and cover or coat it with a metal foil or layer. In a scattering range (free space) determine the $s_\alpha^{(f)}$ for the external resonances, and then scale to find the $s_\alpha^{(1)}$ in the medium of interest. Note that since this is a dual problem the dependence of the pole residues (coupling vectors) on polarization is rotated by $\pi/2$ due to the interchange of electric and magnetic fields.

Of course, this procedure only gives the unperturbed external resonances, i.e. in the limit as $\tilde{\xi} \to \infty$ (or $\epsilon_r \to 0$).

V. General Properties of Internal Resonances

As $\tilde{\xi} \to \infty$ (or $\epsilon_r \to 0$) the target interior V behaves as though S were a perfectly conducting sheet. This gives, as the starting point, the modes of a lossless cavity. Since $\tilde{\xi}$ is finite, loss (damping) is introduced into the corresponding natural frequencies $s_\alpha^{(2)}$, giving these a negative real part. As shown in both the slab and sphere scattering

$$
\begin{array}{lll}
\mathrm{Re}\left[s_\alpha^{(2)}\right] T = -C \epsilon_r^{\frac{1}{2}} + O(\epsilon_r) \text{ as } \epsilon_r \to 0 & & \\
C > 0 & & \\
C \equiv & \text{dimensionless parameter depending on mode and target shape} & \\
T \equiv & \text{some characteristic time associated with propagation through the} & (5.1) \\
 & \text{target (in medium 2, the target medium)} &
\end{array}
$$

There is another small correction to $\mathrm{Im}\left[s_\alpha^{(2)}\right]$ proportional to $-\sqrt{\epsilon_r}\,\sigma_1/\left(\omega_\alpha^{(0)}\,\epsilon_1\right)$, but this is relatively unimportant in many cases of interest in which one is operating in the high-frequency window $\left(\sigma_1/(\omega_\alpha\,\epsilon_1) << 1\right)$.

From an experimental point of view one can take a dielectric target and cover or coat it with a metal foil or layer (as in the previous section). Then through one or more small holes one can couple a small probe, the input impedance of which can be measured as a function of frequency to determine the unperturbed interior resonances. The use of both electric and magnetic probes can be used to sort out mode types. One can also place this covered target with one or more small holes in a scattering range to determine the internal resonant frequencies. Note that the hole locations are important in that a probe should not be placed at a null in the mode of interest (so that one can effectively excite the mode).

To determine the damping of these interior modes the dielectric target can be placed in a medium of known ϵ_1 $(> \epsilon_2)$ for a scattering measurement. In general one will want to know the damping over some range of ϵ_1. This can be accomplished to some degree by variation of the water content of the soil. Thereby the parameters in (5.1) can be determined, noting the asymptotic nature of this formula.

VI. Concluding Remarks

Identifying dielectric targets (insulating) in a lossy external dielectric (such as soil) requires first that the contrast between the two media be sufficiently large that one can detect the presence of the target in a scattering experiment. In some cases (such as by wetting down the soil) one might even increase the contrast for this purpose.

For our present considerations the permittivity of the target is taken as small compared to that of the external medium $(\epsilon_r << 1)$. This leads to an appropriate set of asymptotic results for general target shapes, based in part on canonical geometries (infinite slab and sphere). As such the results are approximate, but useful, for identifying buried dielectric targets.

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